

# Progression in Calculation



# Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

# Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

# Magnitude of Calculations

**Year 1** –  $U + U$ ,  $U + TU$  (numbers up to 20) including adding zero,  $U - U$ ,  $TU - U$  (numbers up to 20) including subtracting zero,  $U \times U$ ,  $U \div U$

**Year 2** -  $TU + U$ ,  $TU +$  multiples of 10,  $TU + TU$ ,  $U + U + U$ ,  $TU - U$ ,  $TU -$  tens,  $TU - TU$ ,  $TU \times U$ ,  $U \div U$

**Year 3** – add numbers with up to three-digits,  $HTU +$  multiples of 10,  $HTU +$  multiples of 100, subtract numbers up to three-digits,  $HTU - U$ ,  $HTU -$  multiples of 10,  $HTU -$  multiples of 100,  $HTU - HTU$ ,  $TU \times U$ ,  $TU \div U$

**Year 4** - add and subtract numbers with up to four-digits,  $ThHTU + ThHTU$ ,  $ThHTU - ThHTU$ , add and subtract decimals with up to two decimal places in the context of money, multiply three numbers together,  $TU \times U$ ,  $HTU \times U$ ,  $TU \times U$ , multiply by zero and one,  $TU \div U$ ,  $HTU \div U$

**Year 5** – add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places,  $ThHTU \times U$ ,  $ThHTU \times TU$ ,  $HTU \times TU$ , multiply whole numbers and decimals with up to three-decimal places by 10, 100 and 1000, divide numbers with up to four-digits by  $U$  (including remainders as fractions and decimals and rounding according to the context)

**Year 6** - add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, multiply numbers with up to four-digits by  $TU$ , multiply numbers with up to two-decimal places by a whole number, divide numbers up to four-digits by  $TU$  (interpreting remainder according to the context), divide decimals up to two-decimal places by  $U$  or  $TU$

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. ... pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

National Curriculum 2014

# Structuring Learning

*Children must have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters.*

Language

Symbols

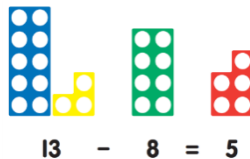
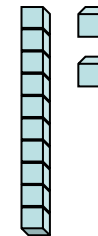
Pictures

Concrete Experiences

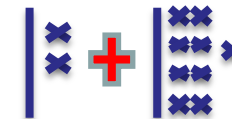
*Haylock and  
Cockburn (2008)*



Active/concrete



Building visual images



13 - 8

Abstract

12 + 19

Communicating  
Mathematically

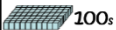
Pattern  
Spotting

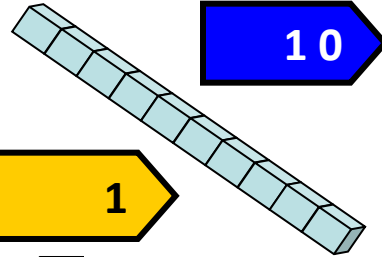
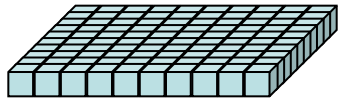
bead string



count stick

place value apparatus

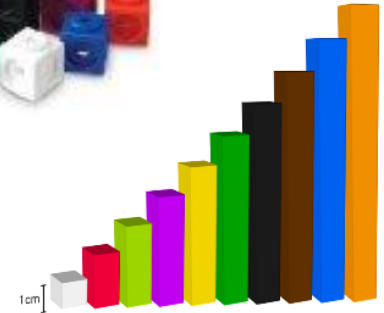
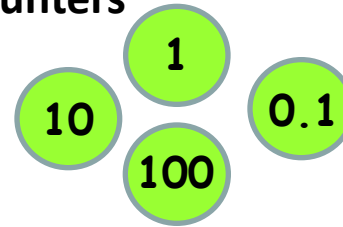
| Hundreds                                                                              | Tens                                                                                  | Units/Ones                                                                           |
|---------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
|  100s |  10s |  1s |
|                                                                                       |                                                                                       |                                                                                      |



Multilink

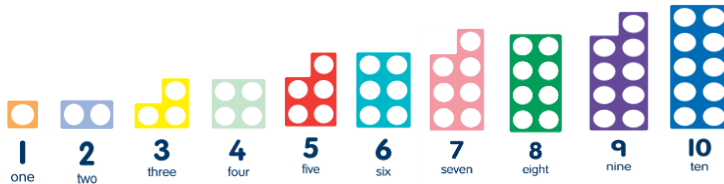


place value  
counters

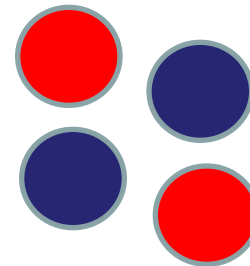
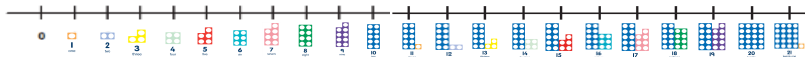


Cuisenaire

Numicon



number line



double sided  
counters

| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
|----|----|----|----|----|----|----|----|----|-----|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
|----|----|----|----|----|----|----|----|----|-----|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

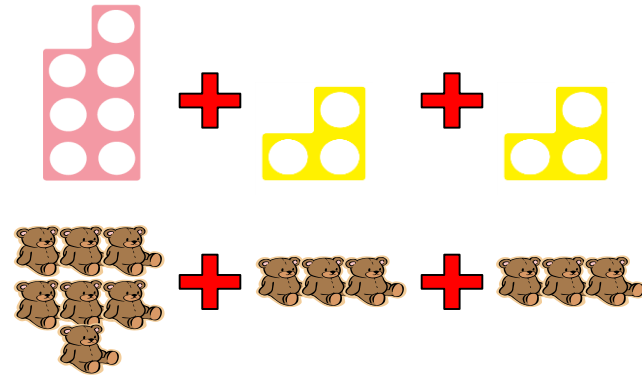
number  
grids  
100 and 200

# Structures of Addition (Haylock and Cockburn 2008)

*Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement*

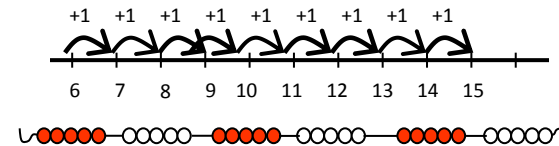
## Aggregation

*Union of two sets  
How many/much altogether?  
The total*



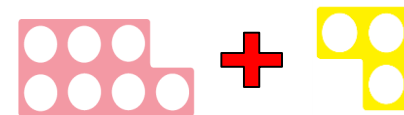
## Augmentation

*Start at and count on  
Increase by  
Go up by*

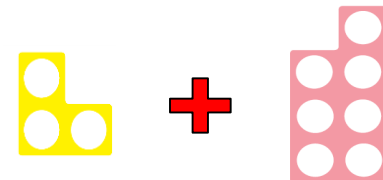


## Commutative law

*Understand addition can be done in any order  
Start with bigger number when counting on  
(Explain to children that subtraction does not have this property)*



*is the same as/equal to (=)*





# Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.  
Addition and subtraction should be taught together.

## End of Year Expectations

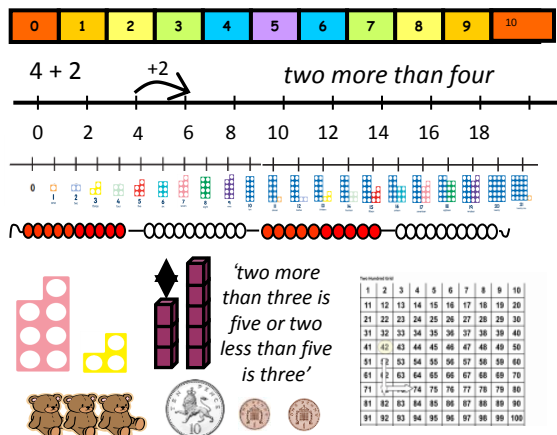
### Year 1

Children must experience combining two, **and then more than two**, groups of objects using counting on and the language of addition e.g. add, plus

Children must experience increasing numbers e.g. what is two more than seven ?

Compare quantities to say how many less and/or how many more

## Possible Concrete and Visual Representations



Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting in groups rather than ones

## Children's Recording

If using Numicon, children could use printed Numicon icons and stick these in - progressing to recording number sentences alongside

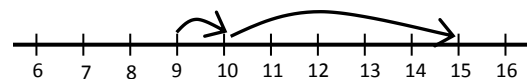
$$1 + 2 = 3$$

Children may record pictorially progressing to recording number sentences alongside

*Example*

$$9 + 6$$

$$9 + 6$$



## Fluency

Count forwards, to and across 100, beginning with 0 or 1 or from any given number

Switch count between tens and ones e.g. 10, 20, 30, 31, 32, 33 ...

Represent and use number bonds up to 20 (establish addition and subtraction as related operations)

Find one more than a number

Find ten more than a number

Count in multiples of 2s, 5s and 10s starting on multiples to highlight pattern recognition

### Year 2

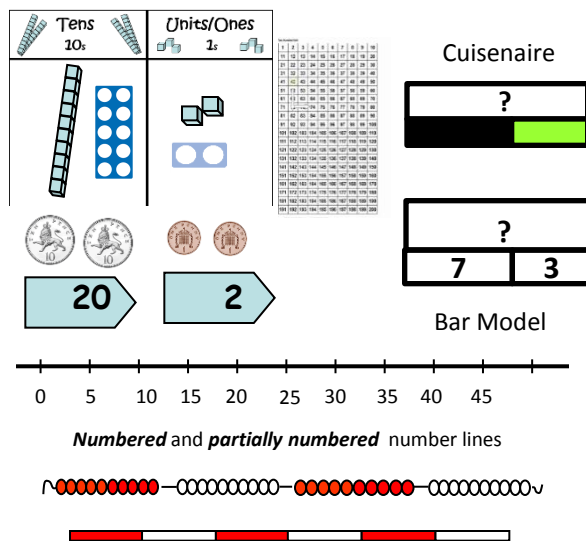
Children should be able to partition numbers in different ways e.g. as  $2+2+2+1$  or  $5+3$  or  $23$  as  $20+3$  or  $10+13$

Children should use concrete objects, pictorial representations and add numbers in different contexts e.g. money, measures

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE** THAN TWO NUMBERS

Children should understand the language of sum

Ensure children understand that addition is commutative (can be done in any order)



Use Numicon, number grids, place value apparatus/Dienes, place value grids, place value cards, Encourage children to partition numbers rather than counting in ones.

Children apply, develop and secure their understanding of place value

Use jottings and record number sentences

| Tens<br>10s | Units/Ones<br>1s |             |
|-------------|------------------|-------------|
|             |                  | 41          |
|             |                  | 28          |
| 40          | 1                |             |
| + 20        | + 8              |             |
| = 60        | = 9              | 60 + 9 = 69 |

Show increasing fluency in deriving pairs of numbers up to 10 and then up to 20

Use knowledge to derive and use number facts up to 100

Add numbers mentally including TU + U, TU + tens, TU + TU, U + U + U

## End of Year Expectations

### Year 3

Add numbers with up to three-digits

(leading to formal written column method)

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** WITH DIFFERING NUMBERS OF DIGITS

Children should partition numbers, up to 1000, in different ways

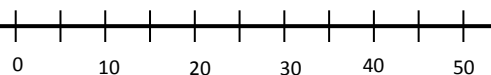
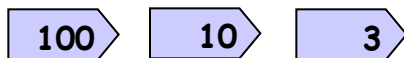
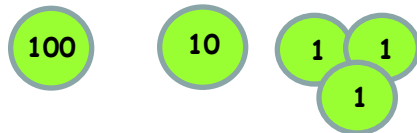
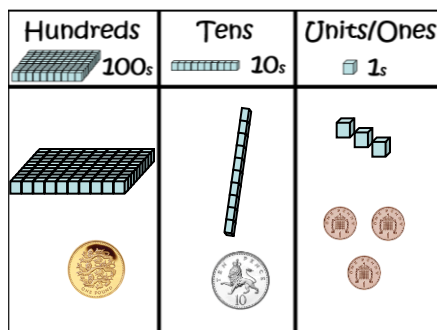
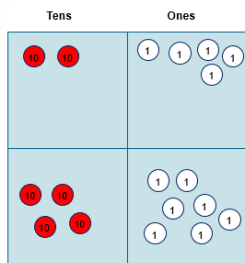
e.g.  $100 + 40 + 6$  or  $100 + 30 + 16$

Solve problems in different contexts including missing number problems

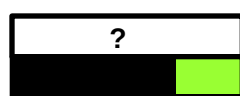
## Possible Concrete and Visual Representations

One Hundred Grid

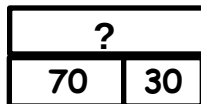
|     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
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| 91  | 92  | 93  | 94  | 95  | 96  | 97  | 98  | 99  | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |
| 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 |
| 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 |
| 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 |
| 151 | 152 | 153 | 154 | 155 | 156 | 157 | 158 | 159 | 160 |
| 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 170 |
| 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 | 180 |
| 181 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 | 190 |
| 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 | 200 |



Partially numbered **and** blank number lines



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Bar Model

## Teacher Modelling/Children's Recording

Children apply, develop and secure their understanding of place value and begin to record in columns

Manipulatives **SHOULD** be used alongside algorithms

Column addition (no exchanging) with up to three-digits

$$\begin{array}{r} 40 + 1 \\ + 20 + 8 \\ \hline 60 + 9 = 69 \end{array}$$

*Expanded recording without exchange*

$$\begin{array}{r} 100 + 40 + 1 \\ + 100 + 20 + 8 \\ \hline 200 + 60 + 9 = 269 \end{array}$$

*Expanded recording*

$$\begin{array}{r} 40 + 3 \\ 20 + 8 \\ \hline 70 + 1 = 71 \\ 10 \end{array}$$

*Expanded recording with exchange*

$$\begin{array}{r} \text{HTU} \\ 141 \\ + 128 \\ \hline 269 \end{array}$$

*Compact (column) recording*

$$\begin{array}{r} 143 \\ + 128 \\ \hline 271 \\ 1 \end{array}$$

Column addition (with exchanging)

$$\begin{array}{r} \text{HTU} \\ 789 \\ + 642 \\ \hline 1431 \\ 11 \end{array}$$

*Compact (column) recording*

$$\begin{array}{r} £7.89 \\ + £6.42 \\ \hline £14.31 \\ 11 \end{array}$$

*Add decimals in the context of money*

## Fluency

Count in ones, tens and hundreds maintaining fluency through varied and frequent practice

Count from 0 in multiples of 4, 8, 50 and 100

Find 10 or 100 more than a number

Mentally add HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers, the answer could exceed 100

Count in 6s, 7s, 9s, 25s and 100s

Find 1000 more than a number

Perform mental calculations with increasingly large numbers to aid fluency

### Year 4

Add numbers with up to four-digits (formal written column method) including numbers with up to two decimal places in the context of money

ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO NUMBERS** INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve two-step problems in different contexts including missing number problems

# Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.  
Addition and subtraction should be taught together.

## End of Year Expectations

### Year 5

Add numbers with more than four-digits and decimals up to three places  
(formal written column method)

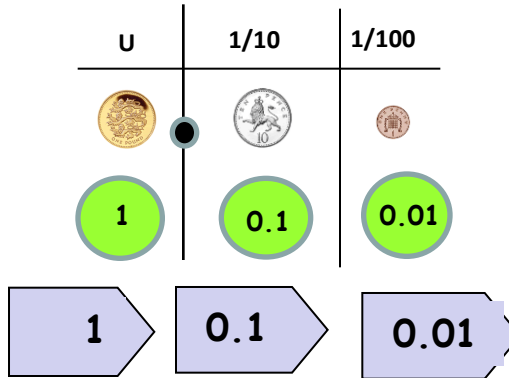
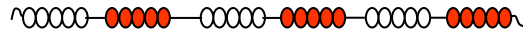
N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO** NUMBERS INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems selecting and justifying methods

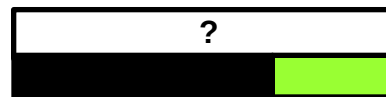
Perform mental calculations with increasingly large numbers

## Possible Concrete and Visual Representations

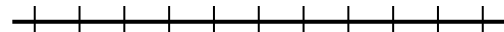
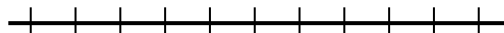
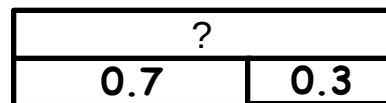
|      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|
| 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.1  | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  |
| 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |



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Bar Model



Partially numbered and blank number lines



## Teacher Modelling/Children's Recording

*Manipulatives could be used alongside algorithms*

$$\begin{array}{r} 2141 \\ + 1128 \\ \hline 3269 \end{array}$$

Column addition (no exchanging)

$$\begin{array}{r} 21.41 \\ + 1.12 \\ \hline 22.88 \end{array}$$

$$\begin{array}{r} 5189 \\ + 3128 \\ \hline 8317 \\ 11 \end{array}$$

Column addition (with exchanging)

$$\begin{array}{r} 51.89 \\ + 3.128 \\ \hline 55.018 \\ 11 \end{array}$$

*Addition with decimals up to three decimal places including in different contexts e.g. money and measures*

## Fluency

Count forwards in powers of ten up to 100000

Count forwards in positive and negative whole numbers through zero

Practise mental calculations with increasingly large numbers

Practise fluency of written methods

### Year 6

Add numbers with more than four-digits and decimals up to three places  
(formal written column method)

N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD **MORE THAN TWO** NUMBERS, INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS

Solve more complex calculations mentally

Solve multi-step problems in contexts, deciding which operations and methods to use and why

Count in tens and hundreds increasing fluency of order and place value

Perform increasingly complex mental calculations and those with increasingly large numbers to aid fluency

# Structures of Subtraction (Haylock and Cockburn 2008)

*Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement*

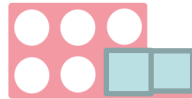
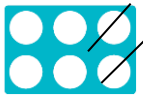
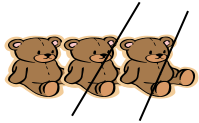
## Partitioning

*Take away*

*... how many left?*

*How many are not?*

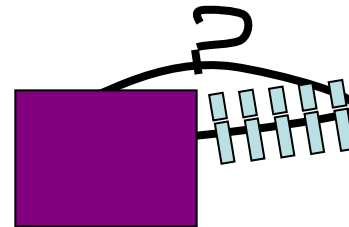
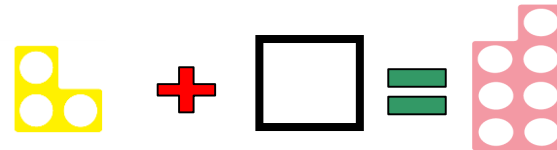
*How many do not?*



## Inverse-of-addition

*What must be added?*

*How many (much) more needed?*



*There are ten pegs  
on the hanger –  
how many are covered?*

## Comparison

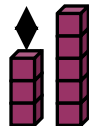
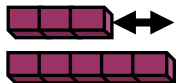
*What is the difference?*

*How many more?*

*How many less (fewer)?*

*How much greater?*

*How much smaller?*



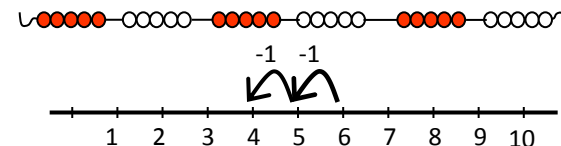
*'two more than three  
is five or two less than  
five is three'*

## Reduction

*Start at and reduce by*

*Count back by*

*Go down by*



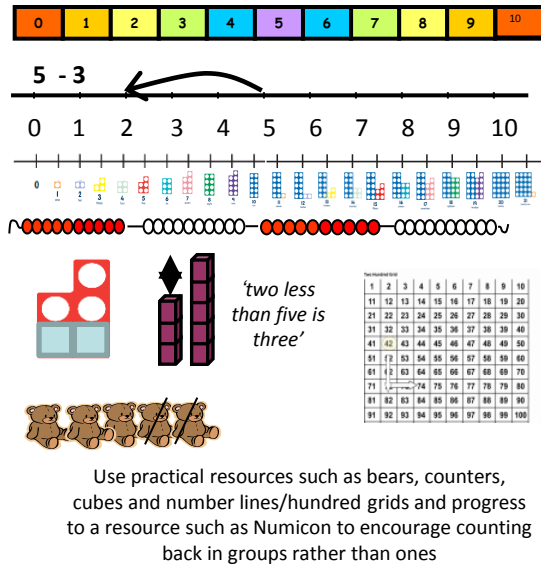
**Addition and subtraction should be taught together.**

## Fluency

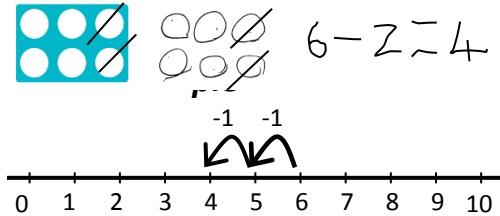
## Year 1

Understand subtraction as taking away  
(What is ... less than ...?)

Compare quantities to say how many less and/or how many more



Children may begin recording  
pictorially progressing to recording  
number sentences alongside



Children could use printed Numicon icons and stick these in, again progressing to recording number sentences alongside

Count backwards (including crossing 100) any given number

Switch count between ones and tens e.g. 33, 32, 31, 30, 20, 10

Represent and use subtraction facts linked to number bonds up to 20 (establish addition and subtraction as related operations)

### Find one less than a number

### Find ten less than a number

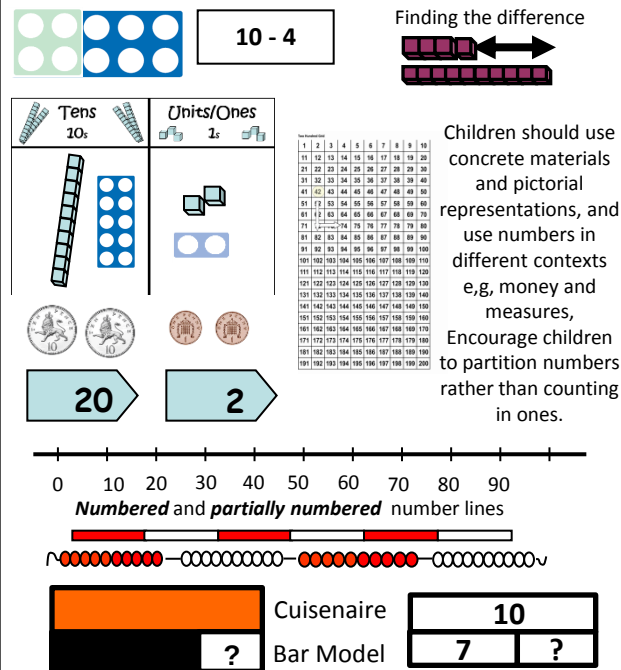
Count back in multiples of 2s, 5s and 10s starting on multiples to highlight pattern

## Year 2

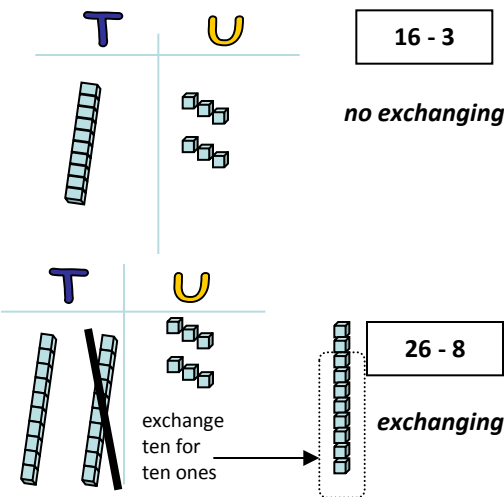
Understand subtraction  
as taking away  
**and** finding the difference

Ensure children understand that subtraction is not commutative (can not be done in any order)

Children should be able to partition numbers in different ways



Children apply, develop and secure their understanding of place value and begin to record using jottings and number sentences



Practise addition and subtraction facts to 20

Show increasing fluency in deriving subtraction facts for numbers up to 10 and then up to 20

Use known facts to 20 to  
derive new facts e.g.  $3 + 7 =$   
 $30 + 70$

Use knowledge to derive  
and use subtraction number  
facts up to 100

# Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.  
Addition and subtraction should be taught together.

## End of Year Expectations

### Year 3

Subtract numbers with up to three-digits

(formal written column method)

Children apply, develop and secure their understanding of place value and begin to record in columns

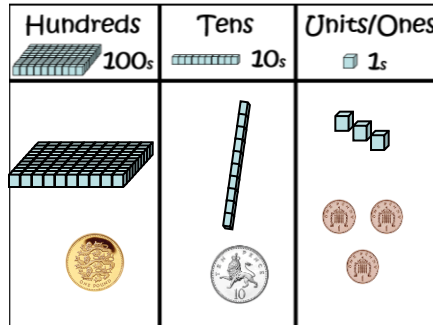
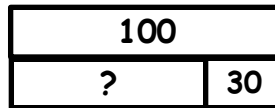
## Possible Concrete and Visual Representations

|     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
| 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  |
| 21  | 22  | 23  | 24  | 25  | 26  | 27  | 28  | 29  | 30  |
| 31  | 32  | 33  | 34  | 35  | 36  | 37  | 38  | 39  | 40  |
| 41  | 42  | 43  | 44  | 45  | 46  | 47  | 48  | 49  | 50  |
| 51  | 52  | 53  | 54  | 55  | 56  | 57  | 58  | 59  | 60  |
| 61  | 62  | 63  | 64  | 65  | 66  | 67  | 68  | 69  | 70  |
| 71  | 72  | 73  | 74  | 75  | 76  | 77  | 78  | 79  | 80  |
| 81  | 82  | 83  | 84  | 85  | 86  | 87  | 88  | 89  | 90  |
| 91  | 92  | 93  | 94  | 95  | 96  | 97  | 98  | 99  | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |
| 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 |
| 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 |
| 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 |
| 151 | 152 | 153 | 154 | 155 | 156 | 157 | 158 | 159 | 160 |
| 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 170 |
| 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 | 180 |
| 181 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 | 190 |
| 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 | 200 |

Cuisenaire



Bar Model



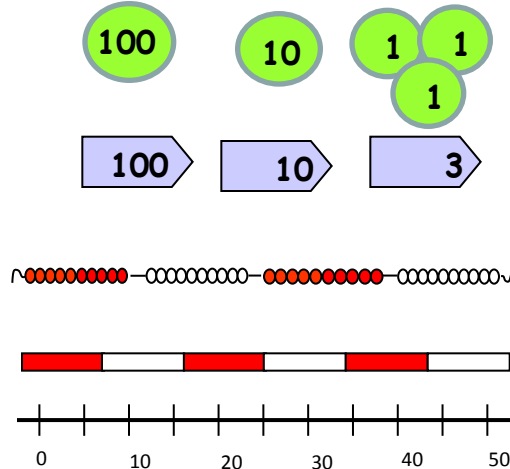
### Year 4

Subtract numbers with up to four-digits

(formal written column method)

Understand subtraction as the inverse of addition

Solve two-step problems deciding upon the appropriate operations and methods and justifying choices made



## Teacher Modelling/ Children's Recording

Children *SHOULD* use manipulatives alongside algorithms to transition between practical and abstract

*no exchange*

$$68 - 23$$

$$60 \quad 8$$

$$20 \quad 3$$

$$40 + 5 = 45$$

*with exchange*

$$63 - 28$$

$$50 \quad 60 \quad 10 + 3$$

$$20 \quad 8$$

$$30 + 5 = 35$$

*Column subtraction (no exchange)*

$$148 - 121$$

$$\begin{array}{r} 100 \quad 40 \quad 8 \\ - 100 \quad 20 \quad 1 \\ \hline \end{array}$$

$$0 + 20 + 7 = 27$$

$$148$$

$$- 121$$

$$27$$

*Column subtraction (with exchange)*

$$723 - 317$$

$$\begin{array}{r} 1 \quad 1 \\ 7 \quad 2 \quad 3 \\ - 3 \quad 1 \quad 7 \\ \hline 4 \quad 0 \quad 6 \end{array}$$

$$723 - 367$$

$$\begin{array}{r} 6 \quad 11 \quad 1 \\ 7 \quad 2 \quad 3 \\ - 3 \quad 6 \quad 7 \\ \hline 3 \quad 5 \quad 6 \end{array}$$

$$\begin{array}{r} 6 \quad 11 \quad 1 \\ \pounds 7 \quad 2 \quad 3 \\ - \pounds 3 \quad 6 \quad 7 \\ \hline \pounds 3 \quad 5 \quad 6 \end{array}$$

Ensure children can solve calculations where zero is a place holder

## Fluency

Count back in ones, tens and hundreds maintaining fluency through varied and frequent practice

Switch count between hundreds, tens and ones e.g 500, 400, 300, 290, 280, 270, 269, 268, 267

Mentally subtract HTU + ones, HTU + tens, HTU + hundreds

Perform mental calculations with two-digit numbers

Find ten and a hundred less than a number with up to three-digits

Count back in 6, 7, 9, 25 and 1000

Count back through zero to include negative numbers

Find 1000 less than a number

Continue to practise mental calculations with increasingly large numbers to aid fluency

# Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.  
Addition and subtraction should be taught together.

## End of Year Expectations

### Year 5

Subtract larger numbers  
(formal written column method)

N.B. ENSURE CHILDREN HAVE THE  
OPPORTUNITY TO SUBTRACT  
DECIMALS WITH DIFFERING NUMBERS  
OF DIGITS

Solve multi-step problems selecting  
and justifying methods

Subtract numbers mentally with  
increasingly large numbers

### Year 6

Subtract multi-digit numbers including  
numbers with up to three decimal places  
(formal written column method)

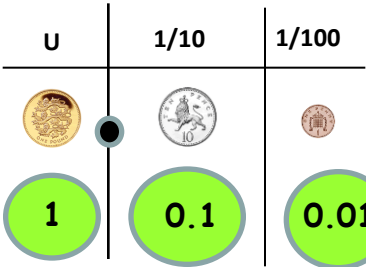
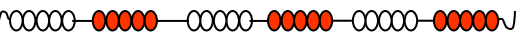
ENSURE CHILDREN HAVE THE  
OPPORTUNITY TO SUBTRACT DECIMALS,  
WITH DIFFERING NUMBERS OF DIGITS

Solve multi-step problems in contexts,  
deciding which operations and  
methods to use and why

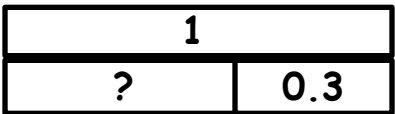
Solve more complex calculations  
mentally

## Possible Concrete and Visual Representations

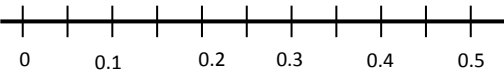
|      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|
| 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.1  | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  |
| 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |



Cuisenaire



Bar Model



## Teacher Modelling/ Children's Recording

*Children might use manipulatives alongside algorithms*

Column subtraction (no exchanging)

$$\begin{array}{r} 13548 \\ - 12128 \\ \hline 1420 \end{array}$$

Column subtraction  
(with exchanging)

$$\begin{array}{r} \overset{2}{1} \overset{13}{3} \overset{11}{4} \overset{1}{2} 3 \\ - 12678 \\ \hline 745 \end{array}$$

*Ensure children can solve calculations  
where zero is a place holder*

$$\begin{array}{r} 1.48 \\ - 1.21 \\ \hline 0.27 \end{array}$$

Column subtraction  
(no exchanging)

Column subtraction  
(with exchanging)

$$\begin{array}{r} \overset{6}{7} \overset{11}{.} \overset{1}{2} 3 \\ - 3.67 \\ \hline 3.56 \end{array}$$

*Subtraction with decimals up to three  
decimal places including in different  
contexts e.g. money and measures*

## Fluency

Count backwards in powers of  
ten up to one million

Count backwards in positive and  
negative whole numbers  
through zero

Practise mental calculations with  
increasingly large numbers

Undertake mental calculations  
with increasingly large  
numbers and more complex  
calculations

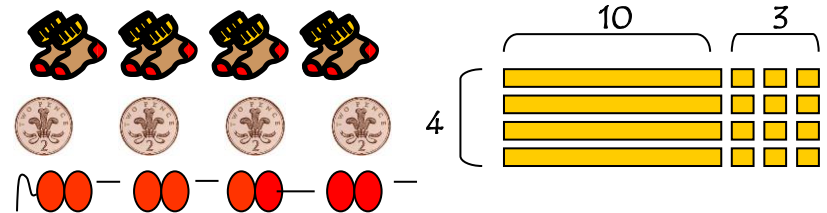


# Structures of Multiplication (Haylock and Cockburn 2008)

*Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement*

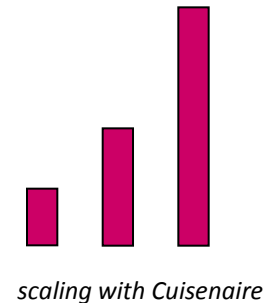
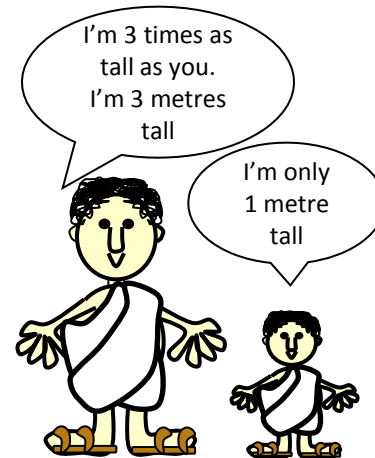
## Repeated addition

*So many lots (sets) of so many*  
*How many (how much) altogether*  
*Per, each*



## Scaling

*Scaling, scale factor*  
*Doubling, trebling*  
*So many times bigger than (longer than, heavier than, and so on)*  
*So many times as much as (or as many as)*



## Commutative law

*Scaling, scale factor*  
*Doubling, trebling*  
*So many times bigger than (longer than, heavier than, and so on)*  
*So many times as much as (or as many as)*

**a x b and b x a are equal**



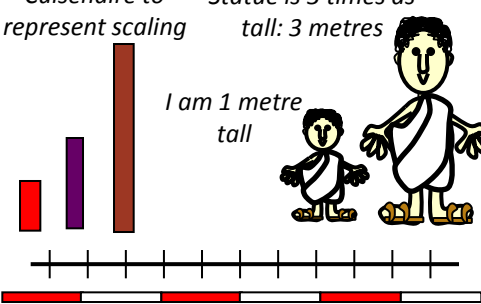
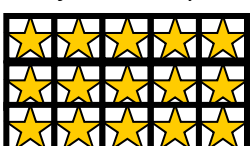
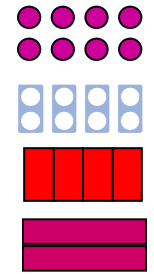
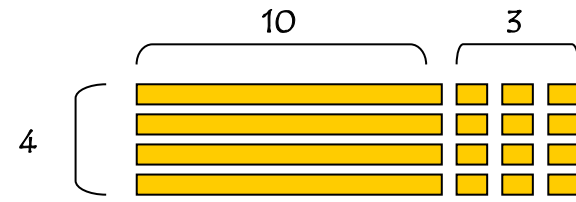
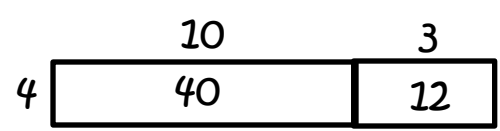
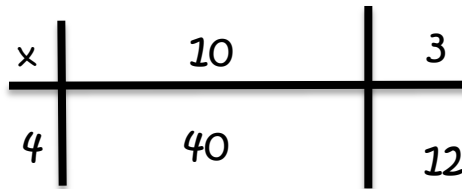
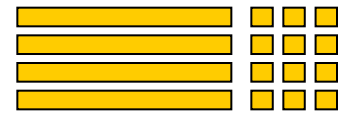
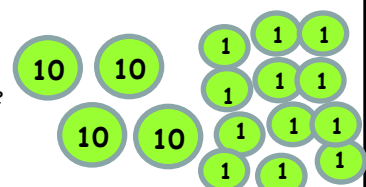
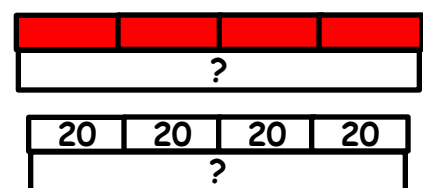
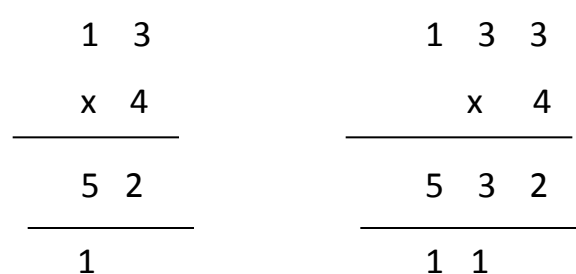
*4 x 2 is the same as/equal to 2 x 4*



**Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly. Multiplication and division should be taught together.**

| End of Year Expectations                                                                                                                                                                                                                                                                                                                                                                                           | Possible concrete and visual representation                                                                                | Children's Recording                                                                                                                                                                                                                                                                                                             | Fluency                                                                                                                                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div>Year 1</div> <p>Solve single step practical problems involving multiplication</p> <p>Use concrete objects, pictorial representations to explore grouping</p> <p>Make connections between arrays, number patterns and counting in twos, fives and tens</p> <p>Double numbers and quantities</p>                                                                                                                | <p>counting in twos</p> <p>arrays- Numicon, Cuisenaire, counters</p> <p>flexible array</p>                                 | <p>Practical only e.g. link to small world</p> <p>Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in</p> <p>four lots of two is eight</p> <p>two lots of four is eight</p> <p>track with cuisenaire</p> | <p>Count in twos, fives and tens from different multiples</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise number patterns</p> <p>Double number and quantities</p>                                                                                                                                    |
| <div>Year 2</div> <p>Understand multiplication as repeated addition</p> <p>Calculate mathematical statements for multiplication within the tables and write them using symbols</p> <p>Understand and solve problems involving arrays</p> <p>Ensure children understand that multiplication is commutative (can be done in any order)</p> <p>Understand that multiplication and division are inverse operations</p> | <p><math>2 + 2 + 2 + 2 = 4 \times 2</math></p> <p>two add two add two add two = four lots of two</p> <p>flexible array</p> | <p>Record practical work as number sentences</p> <p><math>4 \times 2 = 8</math></p> <p><math>2 \times 4 = 8</math></p>                                                                                                                                                                                                           | <p>Count in twos, threes, fives from zero and tens from any number</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise number patterns</p> <p>Introduction to multiplication tables. Practise to become fluent in multiplication facts for 2, 5 and 10</p> <p>Solve multiplication problems mentally</p> |

# Multiplication – multiplication and division should be taught together– refer to structures of multiplication

| End of Year Expectations                                                                                                                                                                                                                                                                                                                       | Possible concrete and visual representation                                                                                                                                                                                                                                                                                                                                                          | Teacher Modelling/Children's Recording                                                                                                                                                                                                                                                                                                                                                                                                                              | Fluency                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div>Year 3</div> <p>Develop reliable written methods</p> <p>Understand and solve scaling problems</p> <p>Solve problems involving multiplication including correspondence</p>                                                                                                                                                                 | <p>Cuisenaire to represent scaling</p> <p>Statue is 3 times as tall: 3 metres</p> <p>I am 1 metre tall</p>  <p>flexible array</p>  <p>arrays</p>  | <p>Children <u>must</u> use manipulatives alongside algorithms</p> <p><math>4 \times 13</math> 'four <u>lots of</u> thirteen'</p>   <p>Expanded methods – grid and area</p>  <p><math>40 + 12 = 52</math></p> | <p>Count from 0 in multiples of 4, 8, 50 and 100</p> <p>Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100</p> <p>Practise mental recall of multiplication tables – 3, 4 and 8x times tables</p> <p>Connect the 2, 4 and 8 times tables using doubling</p> <p>Develop efficient mental methods using commutativity and multiplication facts to derive related facts e.g. <math>4 \times 4 \times 12 = 12 \times 4 \times 5 = 12 \times 20</math></p>                                             |
| <div>Year 4</div> <p>Multiplying three numbers</p> <p>Solve two-step problems</p> <p>Multiplying by 0 and by 1</p> <p>Develop fluency in short multiplication using formal written layout</p> <p>Solve problems involving multiplication including using the distributive law, integer scaling problems and harder correspondence problems</p> | <p><math>4 \times 13</math></p>  <p>place value counters</p>   <p>bar models</p>                                                               | <p>Progressing to developing fluency in short multiplication</p>  <p>Start with digits that are below five so children can practise method without encountering difficulty with multiplication tables</p>                                                                                                                                                                        | <p>Count in multiples of 6, 7, 9, 25 and 1000</p> <p>Recall and use multiplication facts up to <math>12 \times 12</math> with increasing fluency</p> <p>Derive multiplication facts with up to three-digits</p> <p>Recognise and use factor pairs and commutativity in mental calculations</p> <p>Use the distributive law</p> <p>Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. <math>2 \times 6 \times 5 = 10 \times 6</math></p> |

# Multiplication - multiplication and division should be taught together– refer to structures of multiplication

## End of Year Expectations

### Year 5

Multiply decimals with up to three decimal places

Identify multiples and factors including finding all factor pairs of a number, and common factors of two numbers

Solve problems involving all four operations where larger numbers are used by decomposing them into their factors

Multiply whole numbers and those involving decimals by 10, 100 & 1000

Understand and use multiplication and division as inverses including in problems involving missing numbers and balancing equations

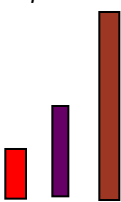
Solve problems involving multiplication and division including scaling by simple fractions

Know and use the vocabulary of prime numbers, prime factors and composite (non-prime)

Recognise and square and cube numbers and associated notation

## Possible concrete and visual representation

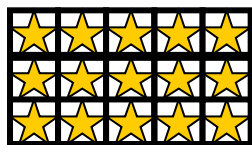
Cuisenaire to represent scaling



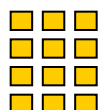
I am 1 metre tall



Statue is 3 times as tall: 3 metres



flexible array



4 x 13

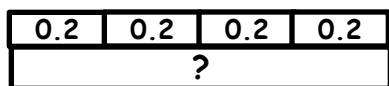
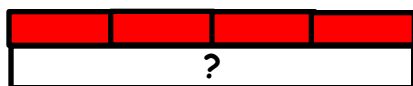
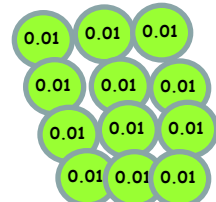


4 x 23

arrays



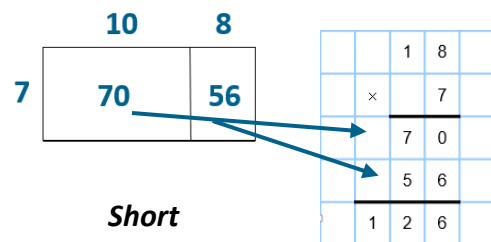
place value counters



bar models

## Teacher Modelling/Children's Recording

Children might use manipulatives alongside algorithms



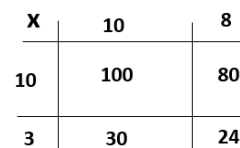
Short multiplication

1 3 2 4

x 6

7 9 4 4

1 1 2



Long multiplication

1 3 2 4

x 2 6

7 9 4 4

2 6 4 8 0

3 4 4 2 4

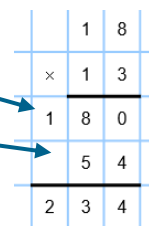
1 1 1

3 . 2 4

x 6

1 9 . 4 4

1 2



3 . 2 4

x 2 6

1 9 . 4 4

6 4 . 8 0

8 4 . 2 4

1 1

## Fluency

Count forwards in steps of powers of 10 from any given number up to 1 000 000

Practise and extend use of formal written method of short multiplication

Apply all multiplication tables frequently. Commit them to memory and use them confidently to make larger calculations

Multiply numbers mentally drawing upon known facts

### Year 6

Multiply numbers up to 4-digit x TU

Multiply numbers with up to two decimal places x whole number

Multiply multi-digit numbers up to four-digits by a two-digit whole number

Multiply single-digit numbers with up to two-decimal places by whole numbers

Solve problems involving all four operations

Undertake mental calculations with increasingly large numbers

Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency

# Structures for Division (Haylock and Cockburn 2008)

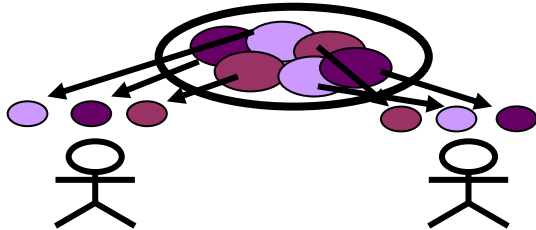
*Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement*

## Equal-sharing

*Sharing equally between  
How many (much) each?*

**6** shared equally by **2**

$$6 \div 2$$

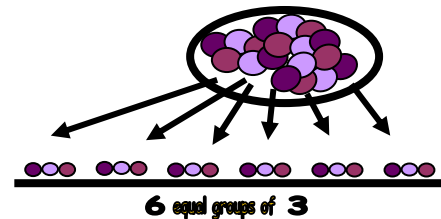


## Inverse of multiplication (Grouping)

*So many lots (sets/groups) of so many  
Share equally in to groups of ...*

$$18 \div 3$$

**18** divided into  
equal groups of **3s**



Divide twelve into equal  
groups of four



= 3

Make 12

Overlay  
groups of  
four

## Ratio structure

*comparison*

*inverse of scaling structure of multiplication  
scale factor (decrease)*

*Barney earns three times more than Fred. If  
Barney earns £900 how much does Fred earn?*

*Jo's journey to school is three times as  
long as Ella's. If Jo walks to school in  
30 minutes how long does it take Ella?*

# Division

Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly.  
Multiplication and division should be taught together.

## End of Year Expectations

### Year 1

Solve single step practical problems involving division

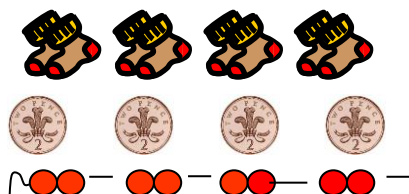
Use concrete objects, pictorial representations

Understand division as grouping and sharing

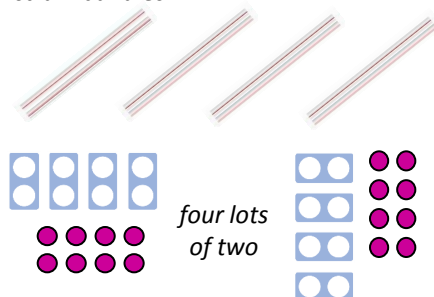
Use the language of 'sharing equally between'

## Possible concrete and visual representation

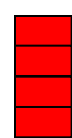
counting in groups of twos



straw bundles



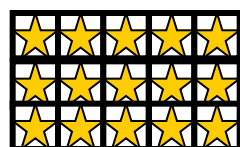
Numicon and counter arrays



Cuisenaire

four lots of two

two lots of four

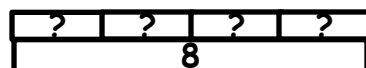


flexible array

doubling



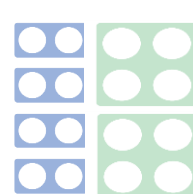
bar models



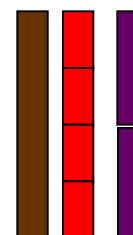
## Teacher Modelling/Children's Recording

Practical only e.g. link to small world

Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in



Eight can be divided into four equal groups of two or two equal groups of four



### Year 2

Solve single step practical problems involving division

Use concrete objects, pictorial representations

Understand division as grouping

Find halves and then quarters

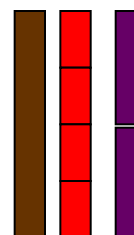
Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete quantities and to arrays

Record as number sentences using  $\div$  and  $=$

$$8 \div 4$$

Eight divided into four equal groups = two in each group

$$8 \div 4 = 2$$



Eight can be divided into four equal groups of two or two equal groups of four

## Fluency

Count in twos, fives and tens from different multiples

e.g. 6, 8, 10, 12 etc

Emphasise patterns

Find simple fractions eg half and quarter, of objects, numbers and quantities

Count back in twos, threes, fives from zero and tens from any number

e.g. 12, 10, 8, 6 etc

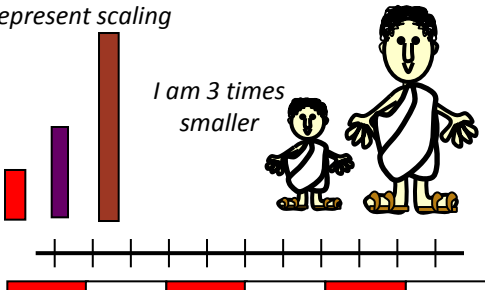
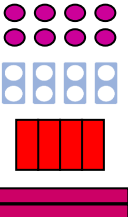
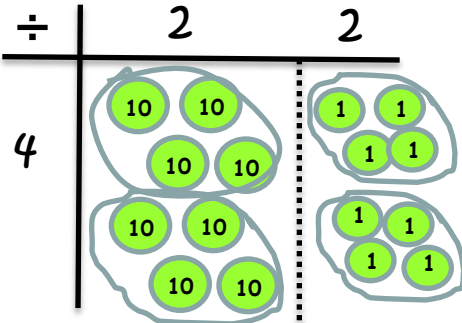
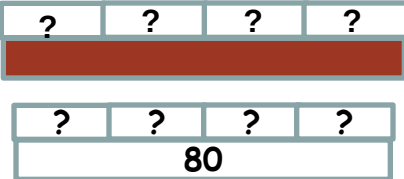
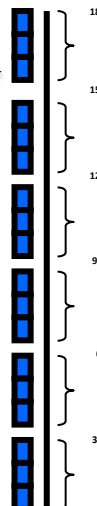
Emphasise patterns

Connect ten times table to place value and five times table to divisions on a clock face

Introduction to multiplication tables. Practise to become fluent in division facts for 2, 5 and 10

Solve division problems involving grouping and sharing

# Division - multiplication and division should be taught together– refer to structures of division

| End of Year Expectations                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Possible concrete and visual representation                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Teacher Modelling/Children's Recording                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Fluency                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div>Year 3</div> <p>Develop a reliable written method for division</p> <p>Solve problems involving missing numbers</p> <p>Solve problems including those that involve scaling</p> <p>Recognise, find and name <math>\frac{1}{2}</math> and <math>\frac{1}{4}</math> of an object, shape or quantity</p> <p>Understand the link between unit fractions and division</p> <p>Connect <math>1/10</math> to division by 10</p> <p>Count in tenths</p>                                | <p>Cuisenaire to represent scaling</p> <p>Statue is 3 metres</p> <p>I am 3 times smaller</p>  <p>arrays</p>   <div> <math>88 \div 4</math> </div>   <p>bar models</p> | <p><i>Children <u>should</u> use manipulatives alongside algorithms</i></p>  <p>Repeated subtraction - chunking</p> <p>Ensure children see/understand the link between grouping on a number line and vertical recording for chunking</p> <div> <math>95 \div 5 = 19</math> </div> <div> <math display="block">\begin{array}{r} 95 \\ - 50 \quad (\underline{10} \times 5) \\ \hline 45 \\ - 25 \quad (\underline{5} \times 5) \\ \hline 20 \\ - 20 \quad (\underline{4} \times 5) \\ \hline 0 \end{array}</math> </div> <div> <b>Fact Box</b> <p><math>2 \times 5 = 10</math></p> <p><math>5 \times 5 = 25</math></p> <p><math>10 \times 5 = 50</math></p> </div>  <div> <math display="block">\begin{array}{r} 321 \\ \div 3 \\ \hline 107 \end{array}</math> </div> <div> <math display="block">\begin{array}{r} 321 \\ \div 3 \\ \hline 107 \end{array}</math> </div> <p>Short division-<br/>no remainders</p> <div> <math display="block">\begin{array}{r} 560 \\ \div 4 \\ \hline 140 \end{array}</math> </div> | <p>Recall and use related division facts for the 3, 4 and 8x tables (Continue to practise other tables)</p> <p>Write and calculate mathematical statements for division using what is known</p> <p>Use division facts to derive related division facts e.g. using <math>6 \div 3 = 2</math> to work out <math>60 \div 3 = 20</math></p> <p>Continue to practise recalling division facts for multiplication tables up to <math>12 \times 12</math></p> <p>Practise mental methods and extend this to three-digit numbers for example <math>200 \times 3 = 600</math> into <math>600 \div 3 = 200</math></p> <p>Use place value, known and derived facts to divide mentally, including dividing by 1</p> <p>Recognise and use factor pairs and commutativity in mental calculations</p> |
| <div>Year 4</div> <p>Become fluent in the formal written method of short division with exact answers when dividing by a one-digit number</p> <p>Divide one- or two-digit numbers by 10 or 100, identifying value of digits as tenths or hundredths</p> <p>Solve two-step problems in different contexts, choosing the appropriate operation, working with increasingly harder numbers including correspondence questions e.g. three cakes shared equally between 10 children</p> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

# Division - multiplication and division should be taught together– refer to structures of division

| End of Year Expectations                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | Possible concrete and visual representation                                                                                                                                                                 | Teacher Modelling/Children's Recording                                                                                                                                                                                                                                                                              | Fluency                                                                                                                                                                                                                                                                                                                                                                                      |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <div>Year 5</div> <p>Identify factors , including finding all factor pairs of a number, and common factors of two numbers</p> <p>Practise and extend the formal written method of short division: numbers up to four-digits by a one-digit number</p> <p>Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding as appropriate for the context</p> <p>Use multiplication and division as inverses</p> <p>Solve problems involving division including scaling down</p> <p>Divide whole numbers and those involving decimals by 10, 100 &amp; 1000</p> | <p><i>Cuisenaire to represent scaling</i></p> <p><i>Statue is 3 metres</i></p> <p><i>flexible arrays</i></p> <p><math>4.8 \div 4</math></p> <p><math>4 \overline{) 1.2}</math></p> <p><i>bar models</i></p> | <p><i>Children might use manipulatives alongside algorithms</i></p> <p>without remainder</p> $560 \div 4$ <p>remainder as a decimal</p> $564 \div 5$ <p>remainder as a fraction</p> $560 \div 24$ <p>long division</p> $560 \div 24$ <p>remainder as a whole number</p> $560 \div 24$ <p>remainder as a decimal</p> | <p>Count backwards in steps of powers of 10 for any given number up to 1 000 000</p> <p>Count backwards with positive/negative whole numbers through zero</p> <p>Practise mental calculation with increasingly large numbers</p> <p>Apply all multiplication tables and related division facts frequently, commit them to memory and use them to confidently to make larger calculations</p> |
| <div>Year 6</div> <p>Divide numbers up to 4-digits by a 2-digit whole number using formal written methods of long division, interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context</p> <p>Divide numbers with up to 2 decimal places by 1-digit and 2-digit whole numbers, initially in practical contexts involving money and measures</p> <p>Understand the relationship between unit fractions and division</p> <p>Recognise division calculations as the inverse of multiplication</p> <p>Solve problems involving division</p>                                                                                                  | <p><math>4 \overline{) 1.2}</math></p> <p><i>bar models</i></p>                                                                                                                                             | <p>remainder as a whole number</p> $560 \div 24$ <p>remainder as a fraction in its lowest form</p> $560 \div 24$ <p>remainder as a decimal</p> $560 \div 24$                                                                                                                                                        | <p>Practise division for larger numbers, using the formal written methods of short and long division</p> <p>Continue to use all multiplication tables and division facts to maintain fluency</p> <p>Perform mental calculations, including with mixed operations and larger numbers</p>                                                                                                      |